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ANALYSIS OF HEAT TRANSFER IN A POROUS COOLED WALL WITH VARIABLE PRESSURE AND TEMPERATURE ALONG THE COOLANT EXIT BOUNDARY

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16. Abstract

Fluid from a reservoir at constant pressure and temperature is forced through a porous wall of uniform thickness. The boundary through which the fluid exits has specified variations in pressure and temperature along it in one direction so that the flow and heat transfer are two-dimensional. The local fluid and matrix temperatures are assumed to be equal and therefore a single energy equation governs the temperature distribution within the wall. The solution is obtained by transforming this energy equation into potential plane coordinates, which results in a separable equation. A technique yielding an integral equation is used to adapt the general solution so that it satisfies the variable-pressure boundary condition. Analytical expressions are given for the normal exit velocity and heat flux along the exit boundary. Illustrative examples are carried out which indicate to what extent the solution is locally one-dimensional.

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SUMMARY

A general analytical solution is obtained for the heat-transfer behavior of a porous cooled wall of constant thickness. The fluid flowing through the porous wall is supplied from a reservoir at constant pressure and temperature. The boundary through which the fluid exits has specified variations in pressure and temperature along it in one direction so that the flow and heat transfer are two-dimensional. The flow within the porous material is assumed to be governed by Darcy's law; that is, the local velocity is proportional to the local pressure gradient. The local fluid and matrix temperatures are assumed to be equal so that a single energy equation governs the temperature distribution within the wall. The solution is obtained by transforming the energy equation into a potential plane where by virtue of Darcy's law the potential is proportional to the local fluid pressure. In this plane the energy equation is separable so that a general solution is obtained. Results are given for the normal velocity and the heat flux along the surface through which the fluid is exiting. Examples are carried out to illustrate the effect of variable pressure and the interaction of variable temperature and pressure along the surface. It is found that in many instances the behavior is close to being locally one-dimensional.

INTRODUCTION

The need to operate some portions of advanced power-producing equipment, engines, and research devices at high temperatures has provided interest in transpiration-cooled media. A porous metallic structure is formed by such means as sintering small metallic particles or rolling together and sintering layers of woven wire cloth. A coolant can then be forced through the porous matrix so that it exits through the boundary exposed to the high-temperature environment. When the porous boundary is being heated by a flowing stream, the transpirant serves a dual purpose. It cools the metallic region as it flows through it; and the exiting coolant pushes away the hot fluid stream, thereby decreasing the heat transfer to the surface. Some possible applications are for cooling turbine blades, components in fusion reactors, arc electrodes, rocket nozzles, high-speed bearings, and vehicles reentering the Earth's atmosphere.

In many instances the coolant exits from the porous material into a region of variable pressure. Two examples of this are the pressure variations resulting from the large acceleration in the throat region of a rocket nozzle and the pressure variation around the surface of a turbine blade. Another example of this occurs when transpiration cooling is used to cool the nose of a body in high-velocity flow. The flow through a porous hemispherical shell in such an environment has been discussed in reference 1; the high pressure at the stagnation point produces coolant starvation in that region.

When the pressure along the exit boundary is variable, the solution for the heat-transfer behavior in the porous medium becomes difficult even for a simple one-dimensional geometry. This is because the flow in the porous medium is two- or three-dimensional and can be quite complicated if the pressure is strongly varying. Since the velocity appears in the energy equation, the solution for the temperature distribution in the porous material then becomes difficult. The resulting equations can be solved numerically; reference 2 gives such a solution for a reentry vehicle nosetip in which both the thickness of the porous material and the external pressure are variable. There is little other literature available for these types of situations.

In the present report an analytical method is developed for determining the porous cooling behavior for a wall of constant thickness with the exit pressure varying along it in one direction. An additional complication is also accounted for which will allow coupling the porous-wall heat transfer with the heat transfer of the external flow. This is that the surface temperature can have a prescribed variation along the surface in the same direction as the pressure variation. The final results relate the heat flux at the high-temperature surface to the imposed surface pressure and surface temperature variations. The relation between surface temperature and heat flux can be used as a thermal boundary condition for the energy equation governing the external flow.

The analysis utilizes the fact that when the fluid within the pores of the material is in very good thermal communication with the surrounding solid matrix (and this situation occurs in many applications), then locally within the material the fluid and matrix temperatures are essentially equal. The heat transfer is then governed by a single energy equation composed of two terms, one representing the energy carried by the flowing coolant and the other the heat conduction within the matrix material.

For the slow viscous flow that occurs in the pores for many porous cooling applications the velocity is governed by Darcy's law, which states that the local velocity is proportional to the local pressure gradient. Since the velocity also satisfies the continuity equation, the pressure in dimensionless form can be regarded as the velocity potential. The porous wall can then be transformed into a potential plane. Since the coolant reservoir is at constant pressure, the adjacent boundary of the medium is a constant potential boundary, while the potential is variable along the coolant exit boundary corresponding to the specified external pressure variation. The reason for considering the porous material in terms of a potential is that by transforming the energy equation into potential plane coordinates the general solution of the equation can be found by using separation of variables. By applying a special technique to adapt this solution to the variable-pressure boundary condition a general method for obtaining solutions to problems of this type is obtained.

ANALYSIS

Governing Equations

Consider the porous wall of thickness δ with constant effective thermal conductivity k_m (based on the entire cross-sectional area rather than on the area of the solid alone) and permeability κ shown in figure 1. There is a fluid with constant density ρ, constant heat capacity C_p, and constant viscosity μ flowing through the wall. Assume that the thermal conductivity of the fluid is very small compared with k_m and that the pore size is so small that Darcy's law holds. Let $\vec{u}$ denote the Darcy velocity (local volume flow divided by entire cross section rather than by open area) of the fluid. We suppose that no changes occur in the direction perpendicular to the x-y plane so the situation is two-dimensional.

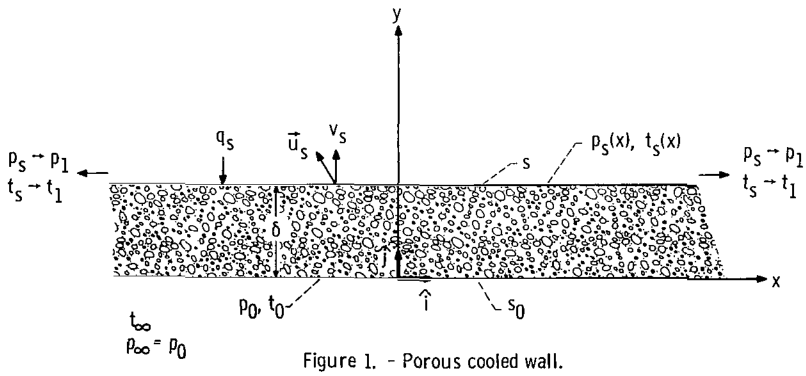


Figure 1. - Porous cooled wall.

Below the porous wall (see fig. 1) there is a reservoir which is maintained at constant pressure and temperature p_0 and t_∞, respectively. The pressure of the fluid along the upper surface of the wall p_s can vary in an arbitrary fashion along the wall as long as the inequality $p_0 > p_s$ is satisfied. Then the fluid flows from the reservoir through the wall and out through the top surface. Since p_0 is constant, the fluid velocity at the lower surface will be in the y-direction. However, the direction of the fluid velocity at the upper surface is not necessarily perpendicular to this surface.

If the thermal communication between the fluid and the matrix is sufficiently good, the local fluid temperature will be approximately equal to the local solid-matrix temperature. We denote this common temperature by t. When these assumptions are made, the heat and mass flow within the porous material are governed by the equations given in reference 3. Thus

$$k_m \nabla^2 t - \rho C_p \vec{u} \cdot \nabla t = 0 \tag{1}$$

$$\vec{u} = -\frac{\kappa}{\mu} \nabla p \tag{2}$$

$$\nabla \cdot \vec{u} = 0 \tag{3}$$

Boundary Conditions

As the fluid in the reservoir at t_∞ approaches the lower boundary s_0 of the porous wall (see fig. 1), the temperature rises to t_0, which varies in an a priori unknown fashion along the x-axis. In most instances of practical interest the fluid velocity is sufficiently high, and the fluid thermal conductivity sufficiently low relative to the thermal conductivity of the solid, that the temperature change from t_∞ to t_0 occurs in a thin fluid region near s_0. This region is therefore assumed to be locally one-dimensional; and since the velocity is perpendicular to the surface (as a result of the pressure being constant along s_0), there is no flow along this thin thermal layer. By applying an energy balance across this thermal layer the boundary conditions at $y = 0$ are found to be

$$\left.\begin{aligned} k_m \frac{\partial t}{\partial y} &= \rho C_p (t - t_\infty) |\vec{u}| \\ p = p_0 &= \text{constant} \end{aligned}\right\} \quad \text{for } y = 0 \tag{4}$$

On the upper surface s at $y = \delta$, we shall suppose that both the temperature and pressure vary in some arbitrarily prescribed manner except that their asymptotic values at $x = -\infty$ are each the same as those at $x = +\infty$. We shall denote this common asymptotic value of the temperature by t_1 and the common asymptotic value of the pressure by p_1. Let $t_2 - t_1$ and $p_2 - p_1$ be the maximum absolute values of the deviations of temperature and pressure away from t_1 and p_1 along $y = \delta$. Then the boundary conditions at $y = \delta$ can be written as

$$t = t_1 + (t_2 - t_1)F(x/\delta) \qquad \text{for } y = \delta \tag{5}$$

$$p = p_1 + (p_2 - p_1)H(x/\delta) \qquad \text{for } y = \delta \tag{6}$$

where the functions F and H are less than or equal to 1 in absolute value and go to zero as x approaches $\pm\infty$.

Dimensionless Equations

The following dimensionless quantities are now introduced:

$$\lambda \equiv \frac{\rho C_p}{2k_m} \frac{\kappa(p_0 - p_1)}{\mu} \tag{7}$$

$$N = \frac{t_2 - t_1}{t_1 - t_\infty} \tag{8}$$

$$P = \frac{p_2 - p_1}{p_0 - p_1} \tag{9}$$

$$\left.\begin{aligned}
X &= \frac{x}{\delta} \\
Y &= \frac{y}{\delta} \\
\varphi &= \frac{p_0 - p}{p_0 - p_1} \\
T &= \frac{t - t_\infty}{t_1 - t_\infty} \\
\vec{U} &= \frac{\mu}{\kappa} \frac{\delta}{(p_0 - p_1)} \vec{u}
\end{aligned}\right\} \tag{10}$$

These dimensionless quantities are substituted into equations (1) to (3) along with the boundary conditions (4) to (6) to obtain

$$\tilde{\nabla}^2 T - 2\lambda \vec{U} \cdot \tilde{\nabla} T = 0 \tag{11a}$$

$$\vec{U} = \tilde{\nabla}\varphi \tag{11b}$$

$$\tilde{\nabla} \cdot \vec{U} = 0 \tag{11c}$$

$$\frac{\partial T}{\partial Y} = 2\lambda \left|\vec{U}\right| T \qquad \text{for } Y = 0 \tag{12a}$$

$$\varphi = 0 \qquad \text{for } Y = 0 \tag{12b}$$

$$T = 1 + NF(X) \qquad \text{for } Y = 1 \tag{13a}$$

$$\varphi = 1 - PH(X) \qquad \text{for } Y = 1 \tag{13b}$$

where

$$\tilde{\nabla} = \hat{i} \frac{\partial}{\partial X} + \hat{j} \frac{\partial}{\partial Y}$$

The porous wall in the dimensionless physical plane is shown in figure 2. When equation (11b) is used to eliminate $\vec{U}$ from the other two equations (11), we obtain

$$\tilde{\nabla}^2\varphi = 0 \tag{14}$$

and

$$\tilde{\nabla}^2 T - 2\lambda\tilde{\nabla}\varphi \cdot \tilde{\nabla}T = 0 \tag{15}$$

Since the surface $Y = 0$ is at constant pressure, the function φ is constant along this line and it follows from equation (11b) that

$$|\vec{U}| = \frac{\partial\varphi}{\partial Y} \qquad \text{on } Y = 0 \tag{16}$$

Hence the boundary condition (12) becomes

$$\frac{\partial T}{\partial Y} = 2\lambda \frac{\partial\varphi}{\partial Y} T \qquad \text{for } Y = 0 \tag{17a}$$

$$\varphi = 0 \qquad \text{for } Y = 0 \tag{17b}$$

Equations (14) and (15), which are related to the flow and energy equations, are to be solved subject to the boundary conditions (13) and (17). Note that since the flow is independent of temperature, the dimensionless pressure φ can be found independently of the temperature by solving equation (14) subject to the boundary conditions (13b) and

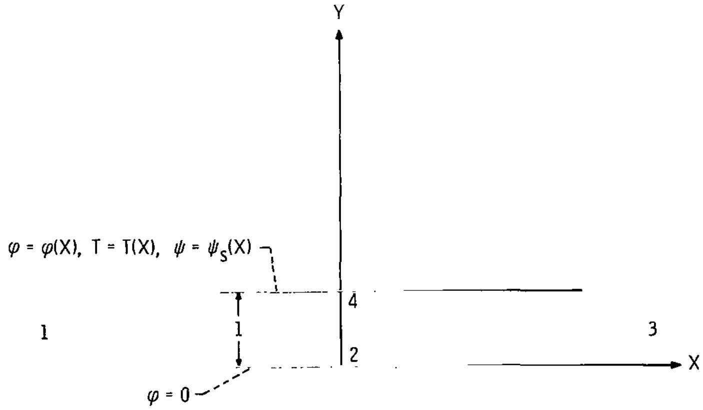


Figure 2. - Dimensionless physical plane.

(17b). Once this has been done, we can substitute the result into equation (15) and solve this equation subject to the boundary conditions (13a) and 17(a).

Solution of Flow Problem

Equation (14) shows that φ satisfies Laplace's equation, and hence there must exist a harmonic function ψ and an analytic function W of the complex variable

$$Z = X + iY \tag{18}$$

such that

$$W = \psi + i\varphi \tag{19}$$

By using the inversion theorem for Fourier transforms it can be verified that the analytic function W whose imaginary part satisfies the boundary conditions (13b) and (17b) is given to within an unimportant real constant by

$$W = Z + \frac{1}{2\pi i}\int_{-\infty}^{\infty} e^{ikZ}\,\frac{C(k)}{\sinh k}\,dk \tag{20}$$

where

$$C(k) = -P\int_{-\infty}^{\infty} e^{-ikX}\,H(X)dX \tag{21}$$

and the Cauchy principle value is to be used in evaluating the integral in equation (20) across the singularity resulting from $\sinh k$ at $k = 0$.

Physically, the change in ψ between any two points is proportional to the volume flow of fluid crossing any curve joining these two points. Hence ψ must vary between $-\infty$ and $+\infty$ as X varies between $-\infty$ and $+\infty$. The mapping given by equation (20) therefore maps the infinite strip in the physical plane which is occupied by the porous wall into a region in the potential plane (W-plane) such as that shown in figure 3 in the manner indicated by the corresponding numbers in figures 2 and 3.

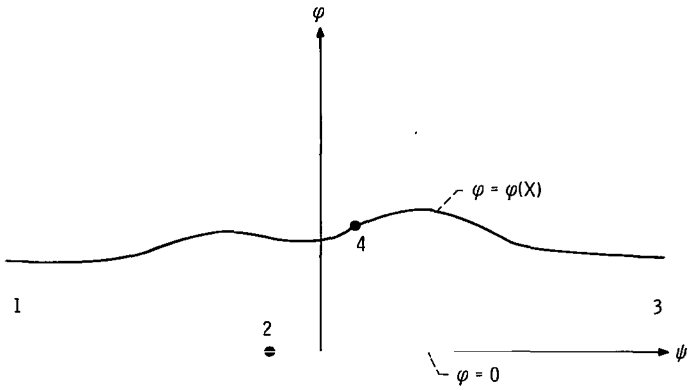


Figure 3. - Complex potential plane (W-plane).

Transformation of Boundary Value Problem for T into Potential Plane

When the independent variables in equation (15) are changed from X and Y to the coordinates ψ and φ of the potential plane, the equation becomes separable. It can be shown from the Cauchy-Riemann equations that (see ref. 4) the first term in equation (15) transforms as

$$\tilde{\nabla}^2 T = \left(\frac{\partial^2 T}{\partial \psi^2} + \frac{\partial^2 T}{\partial \varphi^2}\right)\left|\frac{dW}{dZ}\right|^2$$

The second term is transformed by using the relation (see ref. 3)

$$\tilde{\nabla}\varphi \cdot \tilde{\nabla}T = \frac{\partial T}{\partial \varphi}\left|\frac{dW}{\partial Z}\right|^2$$

Upon substituting these equations into equation (15) we find that

$$\frac{\partial^2 T}{\partial \psi^2} + \frac{\partial^2 T}{\partial \varphi^2} - 2\lambda \frac{\partial T}{\partial \varphi} = 0 \tag{22}$$

Since φ is constant (and equal to zero) at $Y = 0$, the boundary condition (17a) transforms into

$$\frac{\partial T}{\partial \varphi} = 2\lambda T \qquad \text{for} \quad \varphi = 0 \tag{23}$$

These equations can be transformed into a slightly more convenient form by introducing the new dependent variable

$$\Theta = e^{\lambda(1-\varphi)}T \tag{24}$$

Then equations (22) and (23) become

$$\frac{\partial^2\Theta}{\partial\psi^2} + \frac{\partial^2\Theta}{\partial\varphi^2} - \lambda^2\Theta = 0 \tag{25}$$

$$\frac{\partial\Theta}{\partial\varphi} = \lambda\Theta \qquad \text{at } \varphi = 0, \ -\infty \leq \psi \leq +\infty \tag{26}$$

Either by separation of variables (as in ref. 3) or by taking Fourier transforms (with respect to ψ) the general solution to equation (25) which satisfies the boundary condition (26) is found to be

$$\Theta = e^{\lambda(\varphi-1)} + \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{D(\beta)}{\tau_\beta}\left[\frac{(\tau_\beta + \lambda)e^{\tau_\beta\varphi} + (\tau_\beta - \lambda)e^{-\tau_\beta\varphi}}{(\tau_\beta + \lambda)e^{\tau_\beta} + (\tau_\beta - \lambda)e^{-\tau_\beta}}\right]e^{i\beta\psi}\,d\beta \tag{27}$$

where we have put

$$\tau_\beta \equiv \sqrt{\lambda^2 + \beta^2}$$

The first term on the right is the particular solution for uniform pressure and temperature along the boundary $Y = 1$. Hence the integral term gives the deviation from this solution resulting from variable pressure and variable temperature along that boundary.

The function $D(\beta)$ must now be determined so that the boundary condition (13a) is satisfied along the upper boundary $\widehat{341}$ of the region in the potential plane. In order to introduce this boundary condition into the problem, put (fig. 2)

$$\psi_s(X) \equiv \psi(X,1) \tag{28}$$

It now follows from equation (20) that

$$\psi_s(X) = X + \mathcal{R}e\,\frac{1}{2\pi i}\int_{-\infty}^{\infty} e^{ikX}\,\frac{e^{-k}C(k)}{\sinh k}\,dk$$

Since the real part of a complex number Z is $(Z + Z^*)/2$, this becomes

$$\psi_s(X) = X + \frac{1}{4\pi i}\int_{-\infty}^{\infty} e^{ikX}\,\frac{e^{-k}C(k)}{\sinh k}\,dk - \frac{1}{4\pi i}\int_{-\infty}^{\infty} e^{-ikX}\,\frac{e^{-k}C^*(k)}{\sinh k}\,dk$$

Upon changing the variable of integration from k to -k in the last integral and noting that equation (21) implies that $C^*(-k) = C(k)$, we get

$$\psi_s(X) = X + \frac{1}{4\pi i}\int_{-\infty}^{\infty} e^{ikX}\,\frac{e^{-k}C(k)}{\sinh k}\,dk + \frac{1}{4\pi i}\int_{-\infty}^{\infty} e^{ikX}\,\frac{e^{k}C(k)}{\sinh k}\,dk$$

Hence

$$\psi_s(X) = X + \frac{1}{2\pi i}\int_{-\infty}^{\infty} \frac{C(k)}{\tanh k}\,e^{ikX}\,dk \tag{29}$$

Now ψ_s is the value of ψ along the boundary $\widehat{341}$ in the potential plane and is a monotonically increasing function of X. Hence equation (29) can be solved for X as a function of ψ_s to obtain

$$X = X(\psi_s) \tag{30}$$

It is convenient to introduce the functions h and f of ψ_s by

$$h(\psi_s) \equiv H\left[X(\psi_s)\right] \tag{31}$$

and

$$f(\psi_s) = F\left[X(\psi_s)\right] \tag{32}$$

Then equations (13a) and (13b) show that the values of φ and T along the boundary $\widehat{341}$ in the potential plane are given in terms of ψ_s by

$$\left.\begin{aligned} \varphi &= 1 - Ph(\psi_s) \\ T &= 1 + Nf(\psi_s) \end{aligned}\right\} \quad \text{for W on } \widehat{341} \tag{33}$$

It therefore follows from equation (24) that, on this boundary, Θ is given by

$$\Theta = e^{\lambda Ph(\psi_s)}\left[1 + Nf(\psi_s)\right] \qquad \text{for W on } \widehat{341} \tag{34}$$

Hence evaluating equation (27) for W on $\widehat{341}$ and substituting in equations (33) and (34) gives along the boundary $\widehat{341}$

$$2 \sinh \lambda Ph(\psi_s) + Ne^{\lambda Ph(\psi_s)} f(\psi_s)$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\beta\psi_s} \frac{D(\beta)}{\tau_\beta}\left\{\frac{(\tau_\beta + \lambda)e^{\tau_\beta\left[1-Ph(\psi_s)\right]} + (\tau_\beta - \lambda)e^{-\tau_\beta\left[1-Ph(\psi_s)\right]}}{(\tau_\beta + \lambda)e^{\tau_\beta} + (\tau_\beta - \lambda)e^{-\tau_\beta}}\right\}d\beta \tag{35}$$

Equation (35) is a Fredholm integral equation of the first kind which can be solved for $D(\beta)$ once the temperature and pressure distributions F and H (and therefore also f and h) have been specified. However, for numerical purposes it is much more convenient to work with an integral equation of the second kind. In order to obtain an equation of this type we add and subtract the term

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{D(\beta)}{\tau_\beta} e^{i\beta\psi_s} d\beta$$

in equation (35) to obtain

$$2 \sinh \lambda Ph(\psi_s) + Ne^{\lambda Ph(\psi_s)} f(\psi_s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{D(\beta)}{\tau_\beta} e^{i\beta\psi_s}\, d\beta$$

$$+\frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{e^{i\beta\psi_s} D(\beta)}{\tau_\beta}\left\{\frac{(\tau_\beta+\lambda)e^{\tau_\beta}\left[e^{-Ph(\psi_s)\tau_\beta}-1\right]+(\tau_\beta-\lambda)e^{-\tau_\beta}\left[e^{Ph(\psi_s)\tau_\beta}-1\right]}{(\tau_\beta+\lambda)e^{\tau_\beta}+(\tau_\beta-\lambda)e^{-\tau_\beta}}\right\} d\beta$$

Upon taking the Fourier transform of both sides of this equation and interchanging the order of integration, we get

$$\Gamma(\alpha) = D(\alpha) + \frac{1}{2\pi}\int_{-\infty}^{\infty} K(\alpha;\beta)D(\beta)d\beta \tag{36}$$

where we have put

$$\Gamma(\alpha) \equiv \tau_\alpha \int_{-\infty}^{\infty} e^{-i\alpha\psi_s}\left[2\sinh \lambda Ph(\psi_s) + Ne^{\lambda Ph(\psi_s)} f(\psi_s)\right] d\psi_s \tag{37}$$

$$K(\alpha;\beta) \equiv \frac{\tau_\alpha}{\tau_\beta}\,\frac{(\tau_\beta+\lambda)e^{\tau_\beta} I_-(\alpha,\beta)+(\tau_\beta-\lambda)e^{-\tau_\beta} I_+(\alpha,\beta)}{(\tau_\beta+\lambda)e^{\tau_\beta}+(\tau_\beta-\lambda)e^{-\tau_\beta}} \tag{38}$$

and

$$I_\pm(\alpha,\beta) \equiv \int_{-\infty}^{\infty} e^{i(\beta-\alpha)\psi_s}\left[e^{\pm P\tau_\beta h(\psi_s)} - 1\right] d\psi_s \tag{39}$$

Although for most functions h and f it will be necessary to evaluate the integrals (37) and (39) numerically, these integrals are simply Fourier transforms and therefore can be obtained almost instantaneously by using a "fast Fourier transform" computer routine. Equation (36) can then be solved numerically for $D(\beta)$ either directly or by iteration.

The terms appearing in equation (36) are complex. It is sometimes convenient for numerical purposes to replace this equation by a real equation. To this end put

$$E(\alpha) = \frac{1 - i}{2} D(\alpha) + \frac{1 + i}{2} D^*(\alpha) \tag{40}$$

Then $E^*(\alpha) = E(\alpha)$ and this shows that $E(\alpha)$ is real. On the other hand, it follows from equation (27) that Θ will be real only if $D^*(\alpha) = D(-\alpha)$ (see appendix A for verification). Hence

$$E(\alpha) = \frac{1 - i}{2} D(\alpha) + \frac{1 + i}{2} D(-\alpha)$$

and

$$E(-\alpha) = \frac{1 + i}{2} D(\alpha) + \frac{1 - i}{2} D(-\alpha)$$

Therefore

$$D(\alpha) = \frac{1 + i}{2} E(\alpha) + \frac{1 - i}{2} E(-\alpha) \tag{41}$$

Substituting this into the integral in equation (36) gives after changing the integration variable from β to $-\beta$ in the second integral

$$\Gamma(\alpha) = D(\alpha) + \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\frac{1 + i}{2} K(\alpha; \beta) + \frac{1 - i}{2} K(\alpha; -\beta)\right] E(\beta) d\beta \tag{42}$$

Equation (40) can also be written as

$$E(\alpha) = \mathcal{R}e\, D(\alpha) + \mathcal{I}m\, D(\alpha)$$

Hence, upon taking the real part of equation (42) and adding it to the imaginary part, we get after some manipulation

$$\mathcal{R}e\,\Gamma(\alpha)+\mathcal{I}m\,\Gamma(\alpha) = E(\alpha) + \frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\frac{1}{2}\left[K(\alpha;\beta)+K^{*}(\alpha;\beta)\right] - \frac{i}{2}\left[K(\alpha;-\beta)-K^{*}(\alpha;-\beta)\right]\right\}E(\beta)d\beta$$

By further defining

$$\gamma(\alpha) \equiv \mathcal{R}e\,\Gamma(\alpha) + \mathcal{I}m\,\Gamma(\alpha) \tag{43}$$

$$k(\alpha;\beta) \equiv \mathcal{R}e\,K(\alpha;\beta) + \mathcal{I}m\,K(\alpha;-\beta) \tag{44}$$

we get

$$\gamma(\alpha) = E(\alpha) + \frac{1}{2\pi}\int_{-\infty}^{\infty} k(\alpha;\beta)E(\beta)d\beta \tag{45}$$

This is the desired real form of equation (36).

Computation of Surface Fluxes

The physical quantities of principal interest in the problem are the normal velocity of the fluid leaving the upper surface v_s and the conduction heat flux crossing this surface q_s. It is convenient to define the dimensionless normal surface velocity V_s and the dimensionless conduction heat flux Q_s by

$$V_s = \frac{\mu}{\kappa}\frac{\delta}{(p_0 - p_1)}v_s \tag{46}$$

and

$$Q_s = \frac{q_s\delta}{k_m(t_1 - t_\infty)} \tag{47}$$

Then it follows from equations (10) and (11b) that

$$V_s = \left.\frac{\partial \varphi}{\partial Y}\right|_{Y=1} \tag{48}$$

Upon using the Cauchy-Riemann equations this becomes

$$V_s = \left.\frac{\partial \psi}{\partial X}\right|_{Y=1}$$

and by using definition (28)

$$V_s = \frac{d\psi_s(X)}{dX}$$

Substituting equation (29) now shows that

$$V_s(X) = 1 + \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{kC(k)}{\tanh k}\, e^{ikX}\, dk \tag{49}$$

The conduction heat flux into the surface q_s is given by

$$q_s = k_m \left.\frac{\partial t}{\partial y}\right|_{y=\delta}$$

By introducing the dimensionless quantities (10) and (47) this becomes

$$Q_s = \left.\frac{\partial T}{\partial Y}\right|_{Y=1}$$

Hence equation (24) shows that

$$Q_s = e^{\lambda(\varphi-1)}\left(\frac{\partial \Theta}{\partial Y} + \lambda\Theta\frac{\partial \varphi}{\partial Y}\right) \quad \text{at } Y = 1$$

Substituting equation (27) for $\partial\Theta/\partial Y$ in this relation gives

$$e^{\lambda(1-\varphi)}Q_s = \frac{\partial\varphi}{\partial Y}\left\{\lambda\left[e^{\lambda(\varphi-1)} + \Theta\right] + \frac{1}{2\pi}\int_{-\infty}^{\infty} D(\beta)\left[\frac{(\tau_\beta + \lambda)e^{\tau_\beta\varphi} - (\tau_\beta - \lambda)e^{-\tau_\beta\varphi}}{(\tau_\beta + \lambda)e^{\tau_\beta} + (\tau_\beta - \lambda)e^{-\tau_\beta}}\right]e^{i\beta\psi}\,d\beta\right\}$$

$$-\frac{1}{2\pi i}\frac{\partial\psi}{\partial Y}\int_{-\infty}^{\infty}\frac{D(\beta)\beta}{\tau_\beta}\left[\frac{(\tau_\beta + \lambda)e^{\tau_\beta\varphi} + (\tau_\beta - \lambda)e^{-\tau_\beta\varphi}}{(\tau_\beta + \lambda)e^{\tau_\beta} + (\tau_\beta - \lambda)e^{-\tau_\beta}}\right]e^{i\beta\psi}\,d\beta \qquad \text{at } Y = 1$$

Since the Cauchy-Riemann equations show that $\partial\psi/\partial Y = -\partial\varphi/\partial X$, it follows from equations (13b), (24), (28), and (48) that this can be written as

$$Q_s e^{\lambda PH(X)} = V_s\left(\lambda\left\{2\cosh[\lambda PH(X)] + Ne^{\lambda PH(X)}F(X)\right\} + \frac{1}{2\pi}\int_{-\infty}^{\infty} D(\beta)J_-(X;\beta)e^{i\beta\psi_s(X)}\,d\beta\right)$$

$$-\frac{1}{2\pi i}P\frac{dH}{dX}\int_{-\infty}^{\infty}\frac{D(\beta)\beta}{\tau_\beta}J_+(X;\beta)e^{i\beta\psi_s(X)}\,d\beta \tag{50}$$

where we have put

$$J_\pm(X;\beta) \equiv \frac{(\tau_\beta + \lambda)e^{\tau_\beta[1-PH(X)]} \pm (\tau_\beta - \lambda)e^{-\tau_\beta[1-PH(X)]}}{(\tau_\beta + \lambda)e^{\tau_\beta} + (\tau_\beta - \lambda)e^{-\tau_\beta}} \tag{51}$$

Equation (50) can also be expressed in terms of the real quantity $E(\beta)$ by inserting equation (41) and changing the variable of integration from β to $-\beta$ in the integrals involving $E(-\beta)$ to obtain (note that $J(-\beta) = J(\beta)$)

$$Q_s e^{\lambda P H(X)} = V_s(X)\Bigg(\lambda\left\{2\cosh[\lambda P H(X)] + N e^{\lambda P H(X)}\, F(X)\right\}$$

$$+ \frac{1}{2\pi}\int_{-\infty}^{\infty} E(\beta) J_-(X;\beta)\Big[\cos\beta\,\psi_s(X) - \sin\beta\,\psi_s(X)\Big]d\beta\Bigg)$$

$$- P\,\frac{dH(X)}{dX}\,\frac{1}{2\pi}\int_{-\infty}^{\infty} E(\beta)\,\frac{\beta}{\tau_\beta}\,J_+(X;\beta)\Big[\cos\beta\,\psi_s(X) + \sin\beta\,\psi_s(X)\Big]d\beta \tag{52}$$

ILLUSTRATIVE EXAMPLE FOR PRESSURE AND TEMPERATURE VARIATIONS HAVING GAUSSIAN FORM

To illustrate the use of the analytical results some specific solutions will be obtained for some cases where the pressure and temperature vary in an increasing or decreasing manner about a minimum or maximum value. This general type of pressure variation would be encountered, for example, when a slot jet impinges normally against a wall (ref. 5). In this instance the pressure at the surface is a maximum at the stagnation point and drops off monotonically moving away from this point in either direction along the wall. In addition to its specific behavior, the results for the pressure and temperature variations that will be considered will yield some general findings that can be applied to other variations.

The pressure variation along the surface is given by equation (6) and we let

$$H\left(\frac{x}{\delta}\right) = H(X) = e^{-(X-X_p)^2/\sigma_p^2} \tag{53}$$

where σ_p is a parameter that varies the rate at which the pressure changes from its maximum or minimum value p_2 at X_p. For large X the pressure is p_1. The magnitude of the pressure variation along the surface, relative to the pressure drop through the wall, is fixed by the parameter P.

The surface temperature distribution, given by equation (5), will have the same form if we put

$$F\left(\frac{x}{\delta}\right) = F(X) = e^{-(X-X_t)^2/\sigma_t^2} \qquad (54)$$

The temperature variation is thus symmetric about X_t; and its rate of variation is governed by σ_t, which can differ from the parameter σ_p in the pressure distribution. The range of temperature variation along the surface is specified relative to the temperature drop between the coolant reservoir and the hot wall by the parameter N.

To obtain the normal exit velocity from the porous wall the function H(X) is inserted into equation (21). Upon performing the integration the function C(k) is found to be

$$C(k) = -P\sqrt{\pi}\, \sigma_p e^{-ikX_p} e^{-(k\sigma_p/2)^2} \qquad (55)$$

This function is then used in equation (49) to obtain the dimensionless velocity $V_s(X)$ along the surface. The integral can easily be evaluated numerically by using fast Fourier transforms.

After $V_s(X)$ has been found it can be used in equation (52), which gives an expression for the dimensionless heat flux being conducted into the porous wall. This equation also contains the specified pressure and temperature variations as given by P, H(X), N, and F(X). The function $\psi_s(X)$ appearing in the integrals is given by equation (29) in terms of the known function C(k). The function $E(\beta)$ needed to evaluate the integrals in equation (52) is found by solving the integral equation (45) numerically (by using Simpson's rule and then inverting the resulting matrix equations) in which the functions $\gamma(\alpha)$ and the $k(\alpha;\beta)$ are given by equations (43) and (44). The functions Γ and K needed to evaluate equations (43) and (44) are given by equations (37) to (39). The steps in the solution are indicated on the flow chart in figure 4. The integrals in equations (29), (37), and (39) can be easily evaluated numerically by using a fast Fourier transform subroutine. In the solution of equation (45) it is necessary to replace the infinite limits on the integral by finite values. For the numerical results that will be presented, values of ±12 (or ±15 for $\sigma_p = 6$) were found to be sufficiently large to yield results that did not change when the limits were further increased.

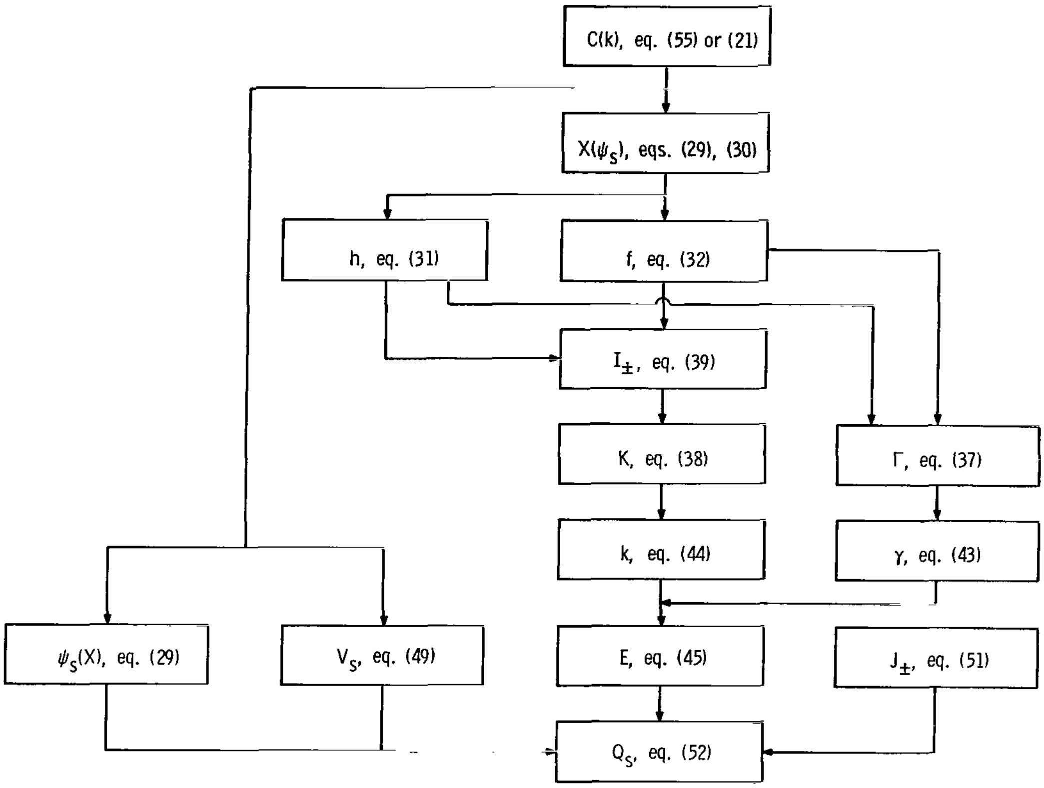


Figure 4. - Equations used to compute surface heat flux.

DISCUSSION

Several illustrative sets of results are now discussed to demonstrate the influence of the parameters which appear in the pressure and temperature variations given in the previous section. In the first few figures there is a variable pressure along the porous surface through which the coolant is exiting, but the surface temperature is constant. Then in the remaining figures the simultaneous interaction of temperature and pressure variations is illustrated.

One of the parameters arising in the analysis is $\lambda = \rho C_p \kappa (p_0 - p_1)/2k_m \mu$ which gives an indication of the relative amounts of energy being transported by the coolant flow and by heat conduction. This can be demonstrated by considering a wall of uniform thickness δ across which there is a uniform pressure drop Δp and a uniform temperature difference Δt. The energy gained per unit area by the coolant flowing through the wall is $v\rho C_p \Delta t$ and the heat flux conducted is $-k_m \Delta t\ \delta$. From Darcy's law $v = -(\kappa/\mu)(\Delta p/\delta)$ so that the ratio of convection to conduction becomes $\rho C_p \kappa \Delta p/k_m \mu$ or 2λ. In what follows, the pressure drop varies with position along the surface so that

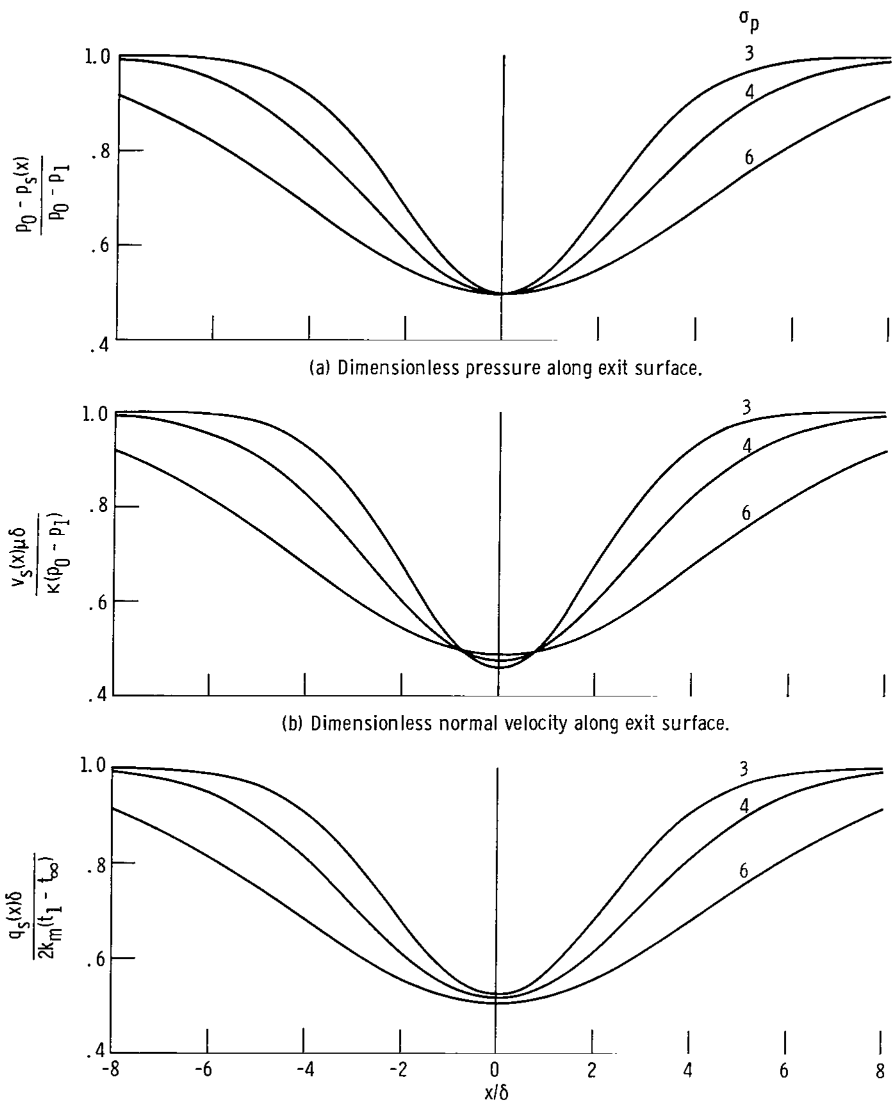
λ, which is based on $p_0 - p_1$, indicates the relative magnitudes of the parameter in the constant-pressure region at large $|X|$. To demonstrate the effect of various pressure and temperature variations the parameter λ will usually be taken as unity. Then the effect of λ will be examined for a variable pressure with the exit surface at uniform temperature and with the exit surface at variable temperature.

The effect of a variable pressure of the form described by equation (53) is shown in figure 5, where the dimensionless imposed pressure difference is plotted in part (a) of the figure. For this example the maximum pressure difference $p_0 - p_1$ occurs at both large positive and large negative X so that the dimensionless pressure drop goes to unity in these regions. For the parameters chosen, the pressure difference decreases to reach one-half of its maximum value at $X = 0$. The parameter σ_p specifies how rapidly the pressure changes along the surface.

The velocity leaving the surface in the direction normal to the surface is given in figure 5(b). The velocity is nondimensionalized so that it is unity in the region of constant pressure at large $|X|$ where the dimensionless pressure drop is also unity. Thus the normalization is such that, if the flow were everywhere locally one-dimensional, the dimensionless velocity would be exactly the same as the dimensionless pressure. The figure shows that this is essentially the case for the most gradual pressure variation shown ($\sigma_p = 6$). For $\sigma_p = 3$, however, the normal velocity decreases at $x = 0$ to somewhat below the locally one-dimensional value of 1/2 as a result of the two-dimensional nature of the flow in the region of increased surface pressure.

When the heat flux being transferred to the fluid exit surface is nondimensionalized as shown in figure 5(c), a locally one-dimensional behavior would produce values identical to the pressure curves. This is evidenced by the fact that the dimensionless heat flux also becomes unity at large $|X|$. It is seen that even for the rapid pressure variations considered, the heat transfer is very close to being locally one-dimensional. This figure shows the heat flux being transferred to the surface when the surface is maintained at a constant temperature. Thus where there is low cooling (low exit velocity), the surface heat flux is low. Even though the normal velocities are a little below 1/2 near $X = 0$, the heat fluxes do not go below 1/2; that is, the local heat flux does not diminish as much as the local normal velocity. This is the result of heat conduction, as will be demonstrated by figure 7. This figure shows that the heat flux does follow the velocity variation more closely when λ is increased, thereby diminishing the relative effect of heat conduction.

Figure 5 showed the effect of the shape of the surface pressure variation on the exit velocity and heat flux along the upper surface. Figure 6 gives similar pressure, velocity, and heat flux curves for a fixed shape of the pressure variation, but with various pressure amplitudes. Shown is the effect of both increased and decreased pressure drop near $X = 0$. There are only very small deviations from locally one-dimensional be-

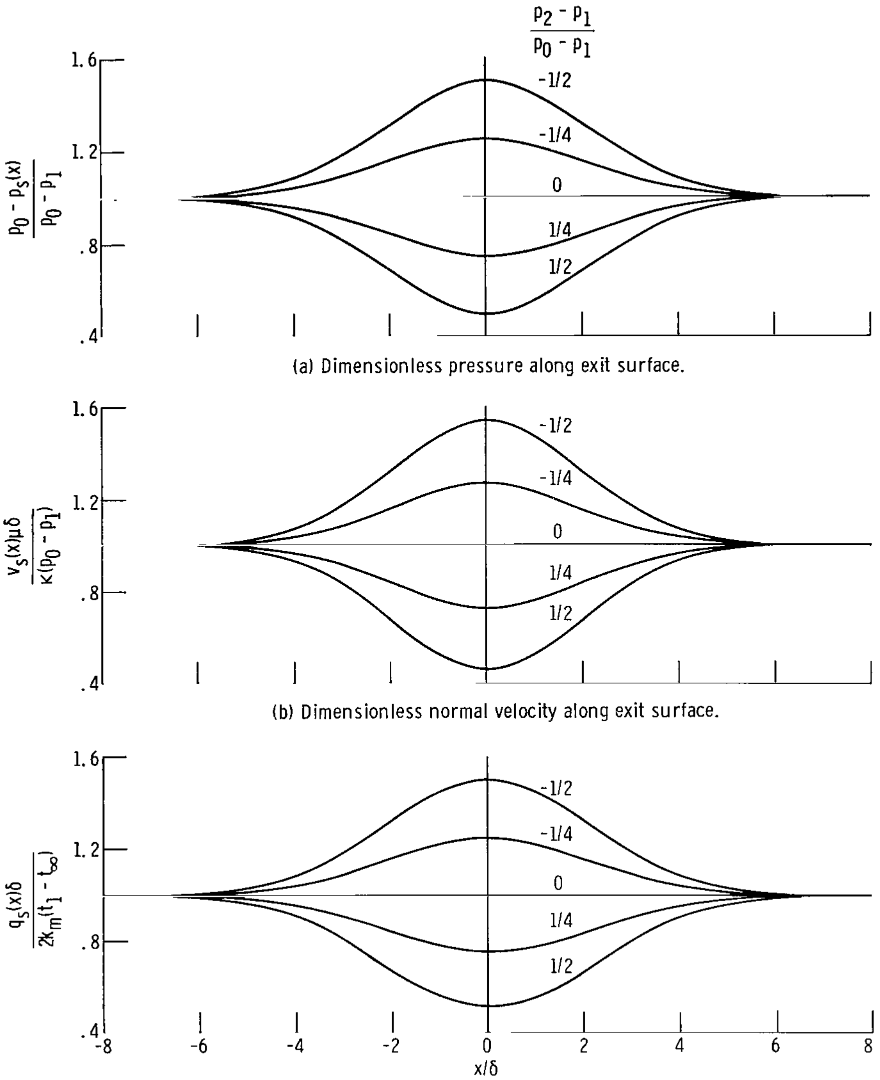


(a) Dimensionless pressure along exit surface.

(b) Dimensionless normal velocity along exit surface.

(c) Dimensionless heat flux along exit surface.

Figure 5. - Effect of various exit pressure distributions on normal velocity and heat flux for exit surface at uniform temperature t_1. $(p_2 - p_1)/(p_0 - p_1) = 1/2$; $\lambda = 1$.



(a) Dimensionless pressure along exit surface.

(b) Dimensionless normal velocity along exit surface.

(c) Dimensionless heat flux along exit surface.

Figure 6. - Heat transfer and normal exit velocity along coolant exit surface for exit surface at uniform temperature t_1. $\sigma_p = 3$; $\lambda = 1$.

havior even though the pressure changes from minimum to maximum values are rapid enough so that they occur only over a distance along the wall equal to about five wall thicknesses.

The parameter $\lambda = \rho C_p \kappa(p_0 - p_1)/2k_m \mu$ is a measure of the relative magnitudes of cooling by the flow and the heat conduction within the medium. If we think of k_m as fixed, then λ changes in proportion to the pressure drop $p_0 - p_1$. Thus a small λ provides decreased flow and a greater influence by the matrix heat conductivity. As a result of the smaller flow associated with a decreased λ the heat flux at the surface would be expected to decrease. In figure 7(c) this effect on the level of the curves has been normalized out by dividing the dimensionless heat flux by λ. Thus the shapes of the curves as a function of λ show only the effect of the relative importance of the matrix conduction. An increased conduction (decreased λ) permits a higher surface heat flux in the low-velocity region, as some of the energy can be redistributed by conduction within the wall interior. However, as shown by the curves, this effect is small for the present case where the exit surface temperature is constant.

The remaining four figures illustrate the effect of simultaneous variations in both surface pressure and temperature. First, consider the special case shown in figure 8 that is given by the curves for $(p_2 - p_1)/(p_0 - p_1) = 0$. In this instance there is a temperature variation along the fluid exit surface as given by the uppermost curve in figure 8(a), but the exit pressure is constant. The temperature variation is given by the uppermost curve in the figure. The temperature difference across the wall (surface to reservoir) at large $|X|$ is $t_1 - t_\infty$, and the distribution shown increases to 1.5 times this value at $X = 0$. Allowing the local surface temperature to increase will permit the local surface heat flux to be larger. Now examine the heat-flux curve for a constant coolant exit pressure $(p_2 - p_1)/(p_0 - p_1) = 0$ in figure 8(b). It is found that this curve is practically the same as the temperature variation, that is, the heat-transfer behavior is almost locally one-dimensional.

For all the cases considered up to now, an individual variation in either the surface pressure or the surface temperature has produced essentially a one-dimensional response in the surface heat flux. As a consequence, when both surface pressure and temperature are varied simultaneously, the heat flux should be approximately the product of the two imposed variations when all quantities are expressed in dimensionless form. This is borne out by the results in figure 8(b) for the nonzero values of $(p_2 - p_1)/(p_0 - p_1)$. In this instance both the temperature and pressure variations are symmetric about $X = 0$.

Figure 9 exhibits the same approximate behavior for a temperature variation that is not symmetric relative to the three pressure variations shown. Thus the region where the surface heat flux can be largest corresponds to the location where the product of pressure drop and surface temperature is the largest.

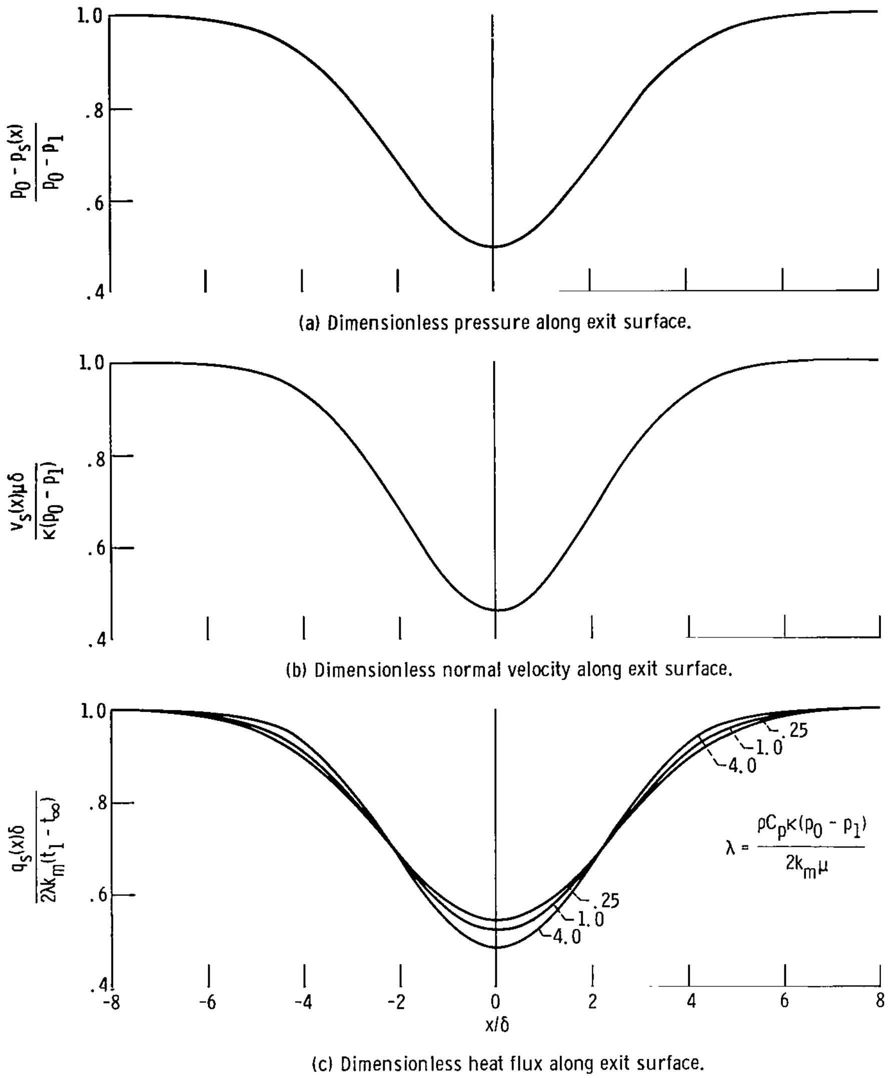
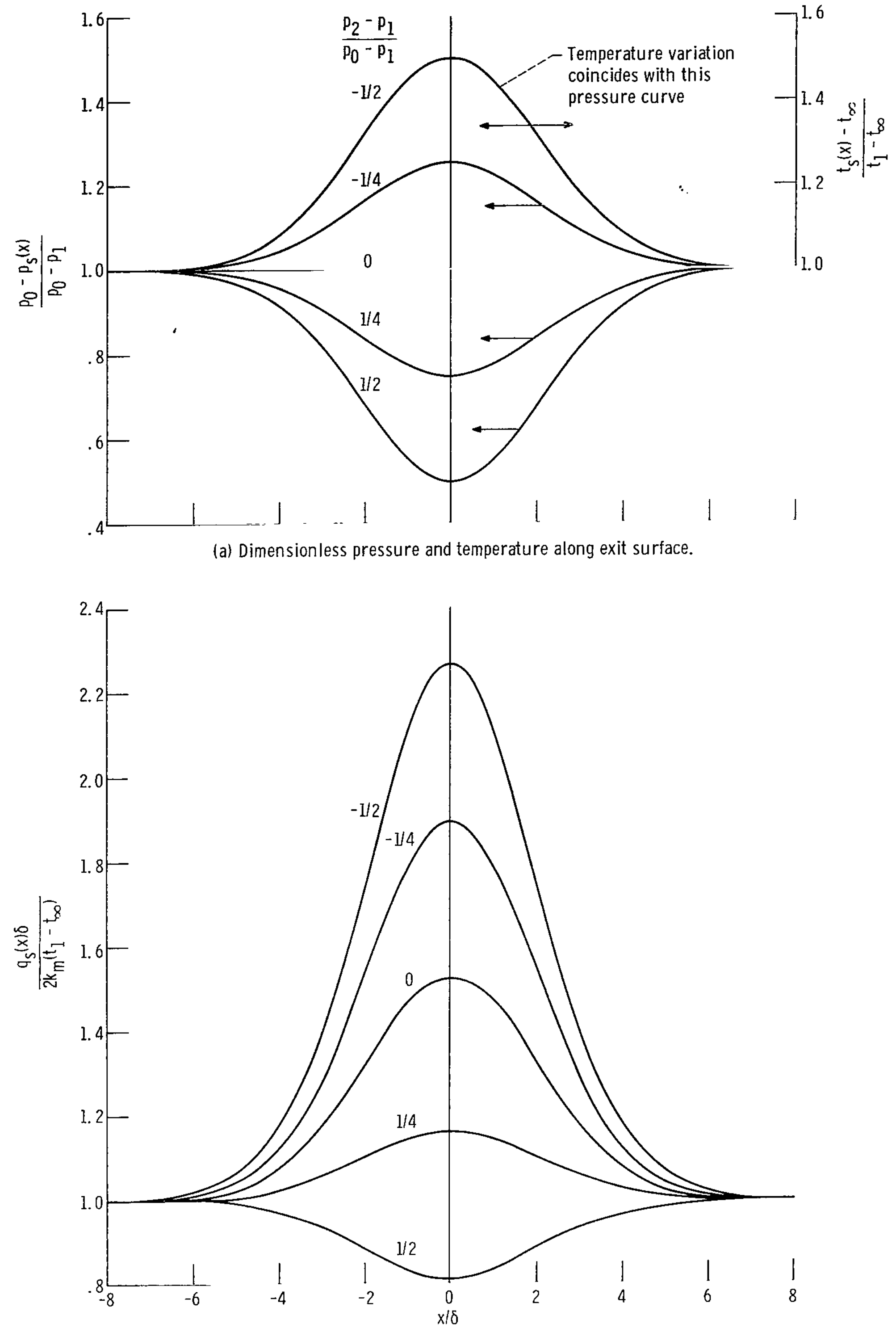


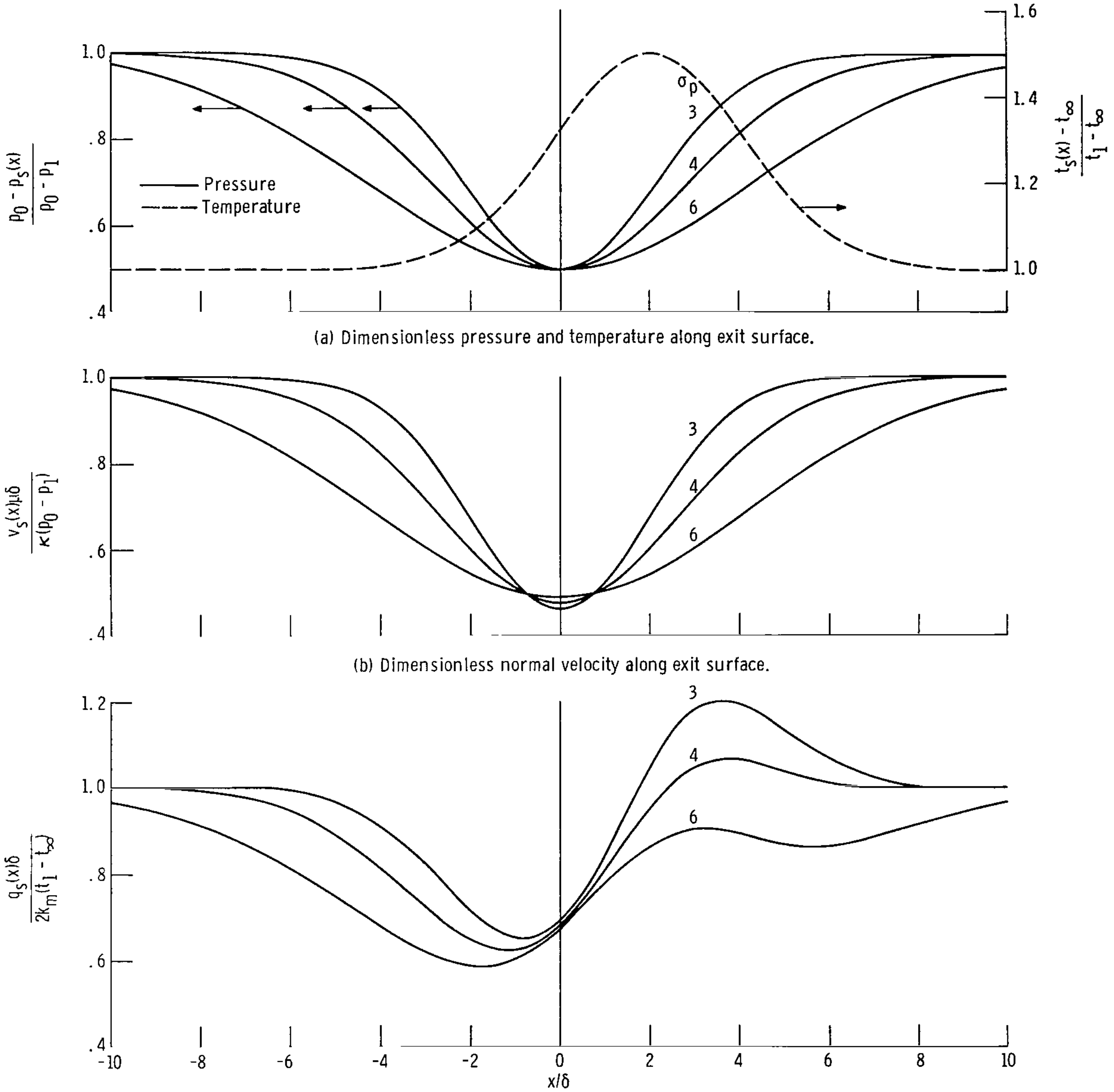
Figure 7. - Heat transfer and normal exit velocity along coolant exit surface for exit surface at uniform temperature t_1 and various values of λ. $\sigma_p = 3$; $(p_2 - p_1)/(p_0 - p_1) = 1/2$.



(a) Dimensionless pressure and temperature along exit surface.

(b) Dimensionless heat flux along exit surface.

Figure 8. - Effect of magnitude of pressure variation on heat flux along exit surface for variable surface temperature. $(t_2 - t_1)/(t_1 - t_\infty) = 1/2$; $\sigma_p = \sigma_t = 3$; $X_p = X_t = 0$; $\lambda = 1$.



(a) Dimensionless pressure and temperature along exit surface.

(b) Dimensionless normal velocity along exit surface.

(c) Dimensionless heat flux along exit surface.

Figure 9. - Heat transfer and normal exit velocity along coolant exit surface for exit surface at variable temperature $t_s(x)$ with three different pressure variations. $(p_2 - p_1)/(p_0 - p_1) = 1/2$; $(t_2 - t_1)/(t_1 - t_\infty) = 1/2$; $\sigma_t = 3$; $X_t = 2$; $\lambda = 1$.

Figure 10 illustrates a situation where the temperature variation is displaced various amounts from $X = 0$, which is the axis of symmetry for the pressure variation. Again as a reasonable engineering approximation the dimensionless heat flux responds as the product of the pressure and temperature variations.

This response is altered somewhat as the parameter λ is varied and this is shown in figure 11. If λ is approximately 1 or larger, heat conduction has little effect relative to convection and the results in figure 11 show a locally one-dimensional response;

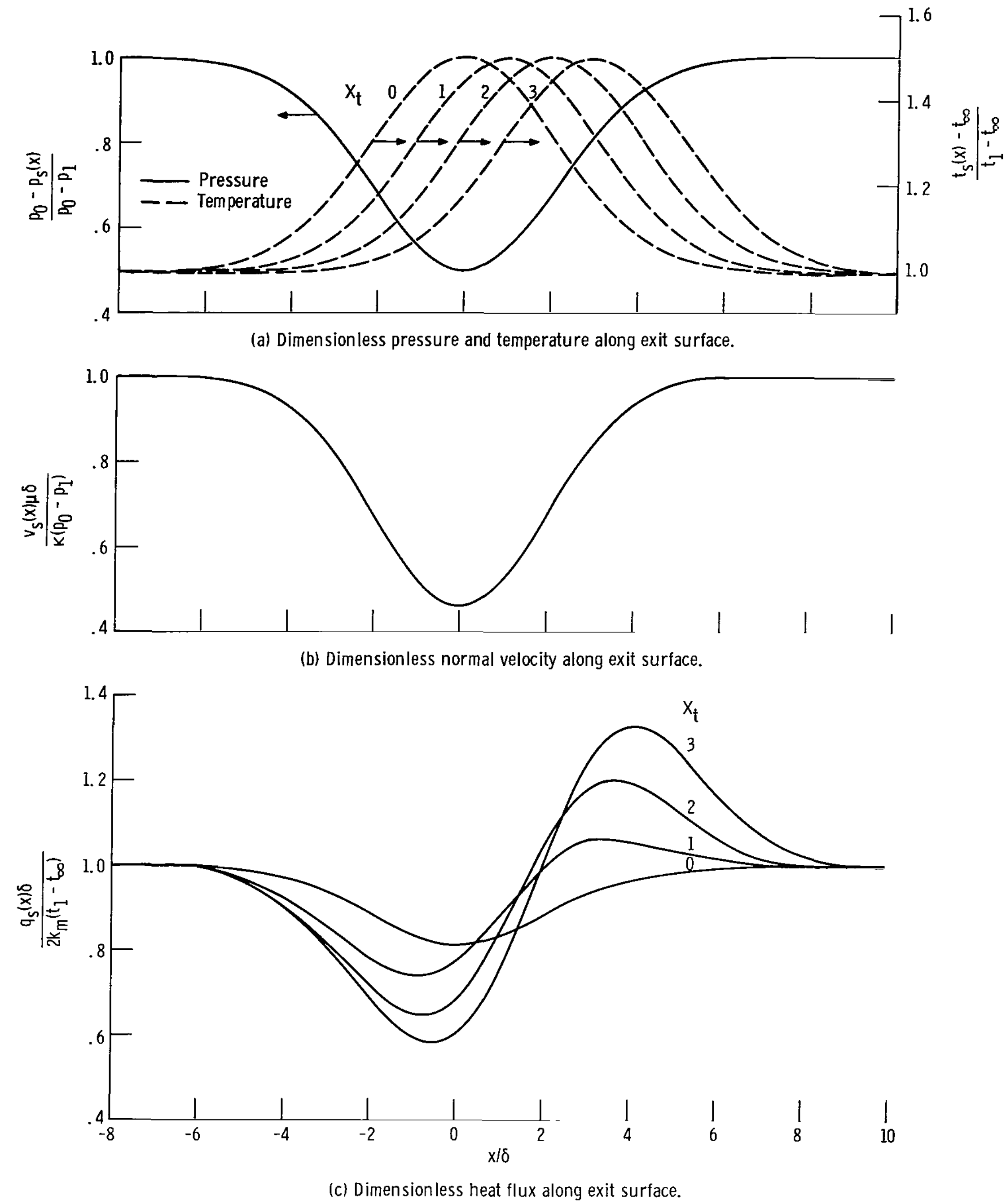
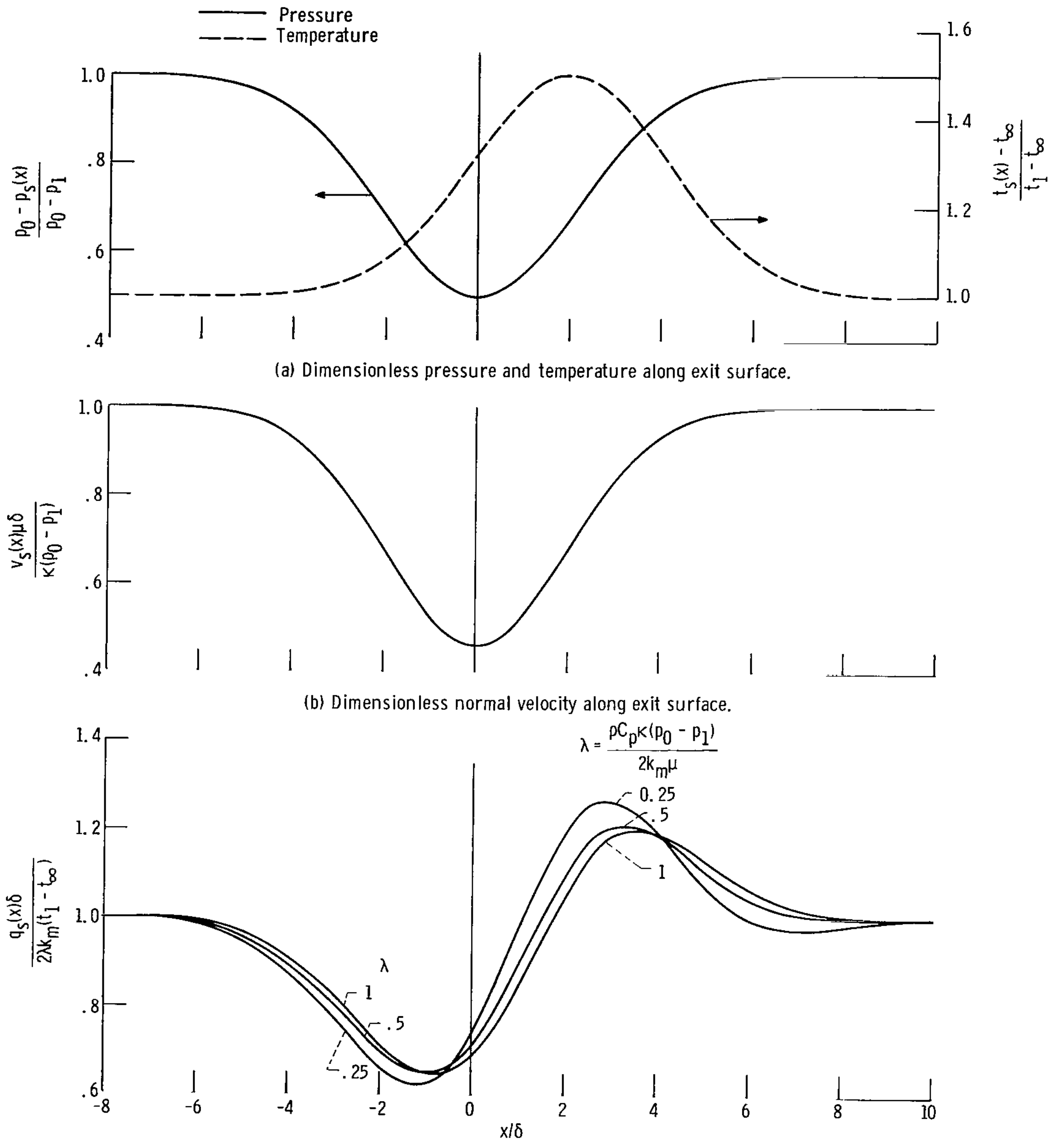


Figure 10. - Effect of various temperature distributions coupled with variable pressure on heat flux along coolant exit surface. $(p_2 - p_1)/(p_0 - p_1) = (t_2 - t_1)/(t_1 - t_\infty) = 1/2$; $\sigma_p = \sigma_t = 3$; $X_p = 0$; $\lambda = 1$.



(c) Dimensionless heat flux along exit surface.

Figure 11. - Effect of λ parameter on heat flux along exit surface for variations in both surface pressure and temperature. $(p_2 - p_1)/(p_0 - p_1) = (t_2 - t_1)/(t_1 - t_\infty) = 1/2$; $\sigma_p = \sigma_t = 3$; $X_p = 0$; $X_t = 2$.

that is, for $\lambda = 1$ the dimensionless $q_s(x)$ curve is essentially the product of the dimensionless $p_s(x)$ and $t_s(x)$ variations. When the conduction is increased relative to the convection (reduced λ), the heat flux is increased in the vicinity of the maximum temperature as a significant amount of heat can now be conducted away.

CONCLUSIONS

An analytical solution has been obtained for the heat-transfer behavior of a porous cooled wall of uniform thickness with variable pressure and temperature along the surface through which the coolant is exiting. The solution was obtained by transforming the energy equation into a potential plane, to obtain a separable equation.

The use of the solution was illustrated by applying it to several combinations of various surface pressure and temperature variations. This revealed a number of interesting features. In all cases the coolant exit velocity followed quite closely the imposed pressure drop across the porous wall. This was true even for situations where the pressure varied so rapidly that it went from its minimum to its maximum value in approximately five wall thicknesses along the surface. For more rapid pressure variations this locally one-dimensional behavior would eventually be violated, but such rapid variations would not usually be expected in practice.

The parameter λ is a measure of the ratio of heat convection to conduction. When $\lambda \geq 1$ (relatively large convection), the heat flux variation was found to essentially follow a variation given by the product of the imposed pressure drop and surface temperature. Hence the maximum q can be tolerated in the local region where both pressure drop and surface temperature are high. This locally one-dimensional behavior is quite important because it justifies the use of a locally one-dimensional analysis for engineering design applications.

For $\lambda < 1$, heat conduction begins to be of some importance and can alter the local one-dimensional dependence of heat flux on the surface temperature variation. For constant surface temperature the properly nondimensionalized surface heat flux changes slowly with λ. With variable surface temperature there is a significant influence of λ, and as conduction becomes more important (λ decreases) the maximum heat flux shifts toward the location of the maximum surface temperature.

The solution relating surface heat flux and surface temperature provides the boundary conditions necessary to couple the heat-transfer behavior of the porous medium with the convection in the external stream.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, September 27, 1971,
132-15.

APPENDIX A

CONDITION ON D(β) TO MAKE Θ REAL

By putting

$$J(\beta) = \frac{1}{\tau_\beta}\left[\frac{(\tau_\beta + \lambda)e^{\tau_\beta \varphi} + (\tau_\beta - \lambda)e^{-\tau_\beta \varphi}}{(\tau_\beta + \lambda)e^{\tau_\beta} + (\tau_\beta - \lambda)e^{-\tau_\beta}}\right] \tag{A1}$$

equation (27) can be written as

$$\Theta = e^{\lambda(\varphi - 1)} + \frac{1}{2\pi}\int_{-\infty}^{\infty} D(\beta)J(\beta)e^{i\beta\psi}\, d\beta \tag{A2}$$

The function $J(\beta)$ is real; and because it depends only on β^2, it satisfies the relation $J(-\beta) = J(\beta)$.

In order that Θ be real the imaginary part of equation (A2) must be zero. Since the imaginary part of a complex number Z can be written as $\mathscr{Im}\, Z = (Z - Z^*)/2i$, it follows from equation (A2) that Θ will be real only if

$$\int_{-\infty}^{\infty} D(\beta)J(\beta)e^{i\beta\psi}\, d\beta - \int_{-\infty}^{\infty} D^*(\beta)J(\beta)e^{-i\beta\psi}\, d\beta = 0$$

By changing the variable in the first integral from β to $-\beta$ we obtain

$$\int_{\infty}^{-\infty} D(-\beta)J(-\beta)e^{-i\beta\psi}\, d(-\beta) - \int_{\infty}^{\infty} D^*(\beta)J(\beta)e^{-i\beta\psi}\, d\beta = 0 \tag{A3}$$

and since $J(-\beta) = J(\beta)$, equation (A3) reduces to

$$\int_{-\infty}^{\infty} \left[D(-\beta) - D^*(\beta)\right] J(\beta)e^{-i\beta\psi}\, d\beta = 0 \tag{A4}$$

Hence Θ will be real only if $D(-\beta) = D^*(\beta)$.

APPENDIX B

SYMBOLS

$C(k)$	function defined in eq. (21)
C_p	specific heat of fluid
$D(\alpha), D(\beta)$	unknown in complex form of integral equation
$E(\alpha), E(\beta)$	unknown in real form of integral equation
F	function in specified temperature distribution (5)
f	defined from F by eq. (32)
H	function in specified pressure distribution (6)
h	defined from H by eq. (31)
$I_{\pm}$	integrals defined in eq. (39)
$\mathcal{I}m$	imaginary part of a function
$\hat{i}, \hat{j}$	unit vectors in X- and Y-directions
$J_{\pm}$	functions defined in eq. (51)
$K(\alpha;\beta)$	kernel defined by eq. (38)
k	Fourier transform variable
k_m	thermal conductivity of porous material
$k(\alpha;\beta)$	real kernel defined by eq. (44)
N	temperature parameter, $(t_2 - t_1)/(t_1 - t_\infty)$
P	pressure parameter, $(p_2 - p_1)/(p_0 - p_1)$
p	pressure
Q_s	dimensionless heat flux, $q_s \delta / k_m (t_1 - t_\infty)$
q_s	conduction heat flux at surface
$\mathcal{R}e$	real part of a function
s, s_0	upper and lower surfaces of porous wall
T	dimensionless temperature, $(t - t_\infty)/(t_1 - t_\infty)$
t	temperature
$\vec{U}$	dimensionless velocity, $\frac{\mu}{\kappa} \frac{\delta}{(p_0 - p_1)} \vec{u}$

$\vec{u}$	Darcy velocity
V_s	dimensionless normal velocity at upper surface, $\frac{\mu}{\kappa} \frac{\delta}{(p_0 - p_1)} v_s$
v_s	normal velocity leaving upper surface
W	complex potential, $W = \psi + i\varphi$
X	dimensionless coordinate, x/δ
Y	dimensionless coordinate, y/δ
x, y	rectangular coordinates
Z	complex variable, $X + iY$
α	transform variable
β	transform variable
Γ	defined in eq. (37)
γ	defined in eq. (43)
δ	wall thickness
Θ	dependent variable defined in eq. (24)
κ	permeability of porous material
λ	parameter, $\frac{\rho C_p}{2k_m} \frac{\kappa(p_0 - p_1)}{\mu}$
μ	fluid viscosity
ρ	fluid density
σ	parameter in pressure and temperature variations, eqs. (53) and (54)
φ	potential, imaginary part of W, $(p_0 - p)/(p_0 - p_1)$
ψ	real part of W
$\tilde{\nabla}$	dimensionless gradient

Subscripts:

p	pressure variation
s	along upper surface
t	temperature variation
0	at lower boundary of wall
1	at large negative and positive x on upper surface

2	value on upper surface having maximum deviation from value at large $\|x\|$
∞	coolant reservoir conditions

Superscript:

*	complex conjugate

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